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Doppler translocation and orbit relaxation techniques

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Satellite Doppler observations made in the translocation mode and processed by 'short arc' and 'orbit relaxation' techniques can lead to very substantial improvements in relative positioning accuracies over those achieved by individual position determinations. Assessing the relative merits of different mathematical solution models is a difficult task which can only be solved by comparing them with an 'absolute' standard.

This has been done by using satellite Doppler observations carried out at nine stations forming part of the Edinburgh–Malvern–Dover precise traverse. This 900 km long traverse, whose stations also form part of the primary triangulation of Great Britain, has full astro-geoidal data and has been reduced and adjusted on the OSGB 77 ellipsoidal datum. The resulting coordinates are accompanied by a corresponding variance–covariance matrix.

The Doppler data obtained along this traverse in the translocation mode were processed by using the newly developed Nottingham 'orbit relaxation' program as well as other commercially available 'short arc' programs.

1. Introduction

One of the major sources of error in geodetic point positioning by the satellite Doppler method is the satellite ephemeris. Positions computed by using the broadcast ephemeris have been found to differ significantly and consistently (e.g. over the United Kingdom and Europe) from those obtained by processing the data with the precise ephemeris (Ashkenazi & Sykes 1978). Moreover, there appear to be disconcerting random jumps in the whole system of satellite coordinates broadcast by the satellites, leading to a significant change in the tracking station coordinates over the period of, say, 1 week (see §4).

One way of overcoming these difficulties is by observing Doppler satellites simultaneously from several tracking stations (the 'translocation mode') and by processing the data with either an 'orbit relaxation' or a 'short arc' technique. It is important to realize that these computational strategies do not increase the accuracy of the geocentric positions of the tracking stations but aim at improving their relative positional accuracies. One such technique was recently developed at Nottingham (see §2.2) and tested successfully.

The tests carried out were not just a series of comparative results obtained by using different software packages, but a succession of direct comparisons with the geodetic coordinates of a high accuracy terrestrial traverse. This precise traverse, connecting Edinburgh, Malvern and Dover, was jointly recomputed by the Ordnance Survey and the University of Nottingham (see §3.)

Translocation observations were carried out from nine of the traverse stations over a period of 3 weeks, each week involving four stations with some of these being occupied for more than 1 week. Nottingham University was helped in this experiment by the Ordnance Survey, Decca

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Survey Limited, the University of Oxford, the Directorate of Overseas Surveys and the Appleton Laboratory of the Science Research Council (Ashkenazi, Dodson, Sykes, Dean & Blanchard 1979, this symposium).

The data were processed by Decca Survey, with JMR's SP-2T translocation program and by Nottingham with the newly developed UNORP program. A full discussion of the comparative results is given in §4. This is followed in §5 by the results obtained from processing translocation data collected at three OSGB stations with the SP-2T, SAGA and UNORP programs.

2. Short arc and orbit relaxation techniques

2.1. *Translocation and simultaneous computations*

The standard method of determining the position of a Doppler satellite tracking station is by processing the observational data with a single point positioning program. The result is expressed by a set of geocentric Cartesian coordinates with respect to the Doppler reference system. The accuracy of the fix is governed to a large extent by the corresponding accuracies of the ephemeris used.

Clearly, the simplest way to reduce some of the effect of the ephemeris errors would be to track the same satellites simultaneously from two or more stations (the 'translocation mode'). The resulting single point determinations will not be more accurate, but the relative positions of the stations are bound to be of a higher standard of accuracy. An example of a pure translocation solution is provided by JMR's SP-2T program.

A better approach is to use the ephemeris only to determine the shape of the orbit, but not its absolute position with respect to the geocentric reference system used. The orbits are then said to be 'relaxed' from their earlier 'fixed' positions, allowing them to translate subject to the purely geometrical constraints imposed by the range–rate observations made simultaneously from several stations. This can be done by using different mathematical models with varying numbers of parameters representing different orbit relaxation strategies (e.g. Nottingham's UNORP program, see §2.2). A variation of this approach is provided by the so-called 'short arc' technique where the ephemeris is generated by a geopotential model, which is relaxed thereafter.

2.2. *Orbit relaxation*

The first orbit relaxation model developed at Nottingham is based on the single point positioning program (UNDAP) developed earlier. The preprocessing and filtering of the data in the two programs is identical (see Ashkenazi *et al.* 1977).

The following is a brief description of the computational procedure used in UNORP (University of Nottingham Orbit Relaxation Program).

(*a*) The data from each station are processed one pass at a time, leading to a set of normal equations with the following unknowns: (i) the frequency offset, $d\Delta f$, for each individual receiver–pass combination, (ii) three position unknowns, dX, dY and dZ, for each station, and (iii) six parameters per orbit which allow it to relax.

(*b*) The frequency unknown is eliminated from each set of normal equations, leaving 9×9 systems of normal equations for each station–pass combination.

(*c*) These are expanded and combined together (in a procedure similar to the Helmert–Wolf

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Block Adjustment method), to form one set of normal equations for each pass. The normal matrices are now of order $6+3n$, where n is the number of stations.

(*d*) The six orbit relaxation parameters are eliminated and the remaining reduced sets of equations combined together. Their solution results in the adjusted coordinates of all the tracking stations, in an arbitrary reference system which is later translated into NWL-10D, the reference system of the broadcast ephemeris.

The six unknown parameters adopted in the mathematical model for relaxing the orbit consist of three translation (X, Y, Z) and three velocity ($\dot{X}$, $\dot{Y}$, $\dot{Z}$) vectors at the point of closest approach of the orbit. Any remaining rank deficiency in the geometry of the model is removed by giving *a priori* 'weights' to the six orbital parameters of each orbit.

2.3. *The short arc method*

The 'short arc' technique is adopted in DBA's SAGA (Short Arc Geodetic Adjustment) program. In contrast to UNORP, where the observation equations for the Doppler cycle counts are set up for successive time intervals, the SAGA program is based on continuously integrated Doppler (CID) counts. This approach lends itself to 'range' rather than 'range rate' observation equations. As a result, SAGA incorporates in its adjustment model four additional unknown parameters, namely a timing bias, a frequency drift, an initial range (at the start of the cycle count) and a factor for the tropospheric refraction correction. These, together with the frequency offset correction and the satellite frequency bias, amount to six known parameters for each station–pass combination (Brown 1970).

The adjustment procedure includes the recomputation of the orbit afresh by using a truncated gravity model, involving between 50 and 75 terms. This is generally sufficient for short arcs not exceeding one-sixth of an orbit.

The relaxation of the orbits is carried out by introducing six weighted unknown parameters for each orbit. These consist of three translation (X, Y, Z) and three velocity corrections ($\dot{X}$, $\dot{Y}$, $\dot{Z}$) applied to the initial state vector of the orbit. Thus the total number of unknowns in the adjustment amounts to

$$3s+6p+6ps+3,$$

where s is the number of stations and p the number of common passes.

3. PRECISE TERRESTRIAL TRAVERSE

To test the accuracy of satellite Doppler positions produced by using the different types of orbit relaxation models, it was decided to use an 'absolute' standard. This was a 900 km precise terrestrial traverse linking the Earlyburn camera station near Edinburgh in Scotland to the Malvern camera station and on from there to Dover on the southeast coast of England (see figure 1). The latter section was part of the Malvern–Graz traverse.

In addition to the values of the geodetic station coordinates which were to be used for the various comparisons, it was essential to have good estimates of the accuracies of this 'absolute' standard. These were obtained from a rigorous least squares adjustment of the traverse and a full covariance analysis.

The traverse consists of 23 stations connected by a series of theodolite observations, EDM distances and 10 Laplace azimuths. The deviation of the vertical was either computed directly from astronomical observations made at the station or was obtained by interpolation from a

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close geoidal section. Distances were reduced on to the reference ellipsoid by using the latest OS 77 Geoid Map. The distances from Malvern to Dover were all measured with the laser geodimeter whereas the majority of those to the north of Malvern were observed with microwave instruments. In the geodimeter distances, frequent instrument calibrations were carried out and the refraction effects largely eliminated by taking meteorological data and vertical angle observations during all the measurements. Similar precautions were also taken with the microwave distances, but frequency checks were not carried out with the same regularity.

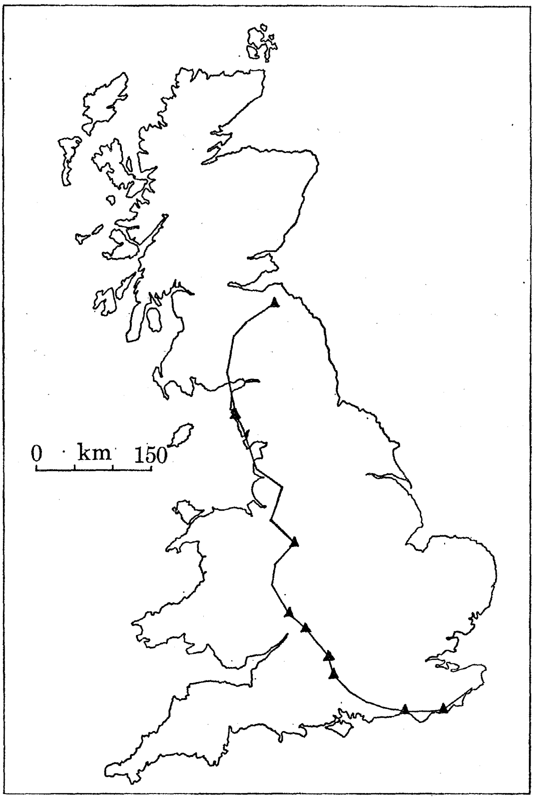

Figure 1. Edinburgh–Malvern–Dover precise traverse. Doppler stations are represented by triangles.

The observations were assigned the following *a priori* standard errors, proposed by the Ordnance Survey: $\pm 2 \times 10^{-6}$ for the geodimeter distances, $\pm 2.5 \times 10^{-6}$ for the microwave distances and $\pm 0.65''$ for both the theodolite directions and the Laplace azimuths. The resulting traverse adjustments gave *a posteriori* standard errors of ± 0.46 m in distance and $\pm 0.27''$ in azimuth (*ca.* ± 0.75 m) for the line joining the two extremities of the traverse. For shorter lines of about 200 km, the corresponding values are approximately ± 0.2 m and $\pm 0.45''$ (*ca.* ± 0.45 m) respectively.

In a series of recent tests by the Ordnance Survey, comparing light wave and microwave measured distances, it was found that there is an unexplained scale discrepancy of about 2.6×10^{-6} between the two types of instrument. This is confirmed by comparisons involving precise ephemeris Doppler derived positions and the OSGB 77 Scientific Network (Ashkenazi, Crane, Williams & Dean 1979, this symposium), which show that this difference is probably due

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to an error in the microwave distances. As a result it was decided to scale all the microwave distances in the traverse up by 2.6×10^{-6}.

4. Doppler–terrestrial comparisons on traverse

4.1. *Observational data*

Satellite Doppler observations were carried out at 9 of the 23 traverse stations over a period of 3 weeks. The stations were occupied four at a time, for a minimum period of 5 days each, leaving 2 days for moving the receivers between stations. One of the stations (Malvern) was occupied permanently for the whole period of 3 weeks and provided the linkage between the three groups of stations.

The Doppler data collected at each station were first processed, in the individual point positioning mode, by using the broadcast ephemeris to assess their quality. They generally appeared to be of good quality, i.e. the Doppler computed positions of the tracking stations agreed with the corresponding values in the traverse (OSGB) system, within the accuracy standards to be expected from broadcast ephemeris processing.

However, the Doppler derived positions of the stations computed from the data collected during the second week showed a consistent shift of 6 m in both X and Y. This significant anomaly cannot be explained. It only involves the second week of observations, and it only occurs when the data are reduced in conjunction with the broadcast ephemeris. Considering that the processing of the same data with the precise ephemeris produced good results throughout the 3 week period, one can only conclude that the origin of the broadcast ephemeris framework appears to have moved by 6 m in both the X and Y directions during the period from 18 to 23 June 1978. However, as the data were to be further processed in the orbit relaxation mode, this anomaly did not present an obstacle, with the exception of the data at one of the stations, which were discarded.

4.2. *Orbit relaxation with the use of broadcast ephemeris*

After 'cleaning', the data were subjected to an adjustment by the orbit relaxation method with the UNORP program. The resulting coordinates were then brought into an arbitrary but consistent system through the common station, Malvern. This was followed by a further translation of the absolute coordinates by an amount required to move their centre of gravity to that of the corresponding (broadcast ephemeris derived) single point positions.

The resulting coordinates are in the broadcast ephemeris system. However, considering that this system has been found to differ significantly from that of the precise ephemeris, one could transform these coordinates by using empirically derived transformation parameters. This can be done in one of the following two ways (Ashkenazi & Sykes 1978; Ashkenazi, Gough & Sykes 1979, this symposium): (*a*) a translation of +3 m in X and −5 m in Z; (*b*) a translation of −3 m in X, −11 m in Z and a scale of $+1.3 \times 10^{-6}$. Lastly, these (pseudo-precise ephemeris) coordinates should be corrected for known longitude and scale bias, the latest recommended values being $\pm 0.8''$ and -0.4×10^{-6} respectively (Hothem *et al.* 1978; Anderle 1978). The Doppler derived coordinates are then ready for comparison with the corresponding terrestrial Traverse coordinates in the OSGB system. One can carry out chord distance comparisons with the Doppler coordinates as they stand. However, for comparisons involving spheroidal distances and azimuths, one has further to transform the Doppler geocentric Cartesian coordinates into

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the traverse (OSGB) system, before turning the resulting Cartesian coordinates into geodetic latitudes and longitudes. The required three translation parameters that were computed by a best fit solution between the individual Doppler positions (derived by using the precise ephemeris) and the corresponding traverse coordinates of the nine stations are −369.9 m in X, +111.7 m in Y and −429.2 m in Z.

The results of these comparisons are listed in table 1 for Doppler data processed with the UNORP program and in table 2 for the SP-2T program. These are given in terms of the mean values of the differences from the Doppler to the traverse system. The results in the first three columns refer to the mean values of the absolute differences of spheroidal distances (in metres) for lines between all possible combinations of stations observed in each of weeks 1, 2 and 3 respectively. Column 4 corresponds to lines between all possible combinations (36) of the nine stations.

TABLE 1. MEAN VALUES OF DOPPLER 78 – TRAVERSE OSGB SCALE AND AZIMUTH DIFFERENCES FOR UNORP PROCESSED DATA

	spheroidal distances/m				lines greater than 300 km	
model	week 1	week 2	week 3	all (36)	chord (parts/10⁶)	sph. az.
raw broadcast	0.29	0.45	0.90	0.63	+0.86	+1.33″
pseudo-precise, trans. only	0.29	0.39	0.86	0.62	+0.86	+1.32″
pseudo-precise, trans. + scale	0.35	0.15	0.71	0.62	−0.44	+1.36″

TABLE 2. MEAN VALUES OF DOPPLER 78 – TRAVERSE OSGB SCALE AND AZIMUTH DIFFERENCES FOR SP-2T PROCESSED DATA

	spheroidal distances/m				lines greater than 300 km	
model	week 1	week 2	week 3	all (36)	chord (parts/10⁶)	sph. az.
raw broadcast	2.42	7.13	0.94	5.04	11.99	−0.57″
pseudo-precise, trans. only	2.54	7.07	0.94	5.01	11.99	−0.58″
pseudo-precise, trans. + scale	2.39	6.83	1.08	4.88	10.69	−0.54″

The mean values of chord distance differences (in parts/10^6) and spheroidal azimuth differences (in seconds of arc), both for lines over 300 km in length, are listed in columns 5 and 6 respectively.

An examination of tables 1 and 2 leads to the following conclusions:

(*a*) The UNORP program seems to produce consistent relative accuracies of the order of 30 cm in weeks 1 and 2 (where the distances involved are up to 300 km in length).

(*b*) The corresponding average value for all three weeks (where the distances of the 36 lines vary from 30 to 600 km) is about 60 cm. This value cannot be entirely attributed to errors in either the Doppler system or the UNORP processing, as the covariance analysis of the traverse shows a reliability of no better than 50 cm in scale along its full length. This may be the reason for the larger than average discrepancies in week 3 (where most of the distances are greater than 300 km).

(*c*) There seems to be a consistent orientation difference between Doppler derived azimuths and traverse (OSGB) azimuths, exceeding 1″.

(*d*) For long (over 300 km) lines, there seems to be very little scale difference between Doppler and traverse (OSGB) distances, particularly when the former are corrected for the known scale bias in the broadcast ephemeris (over Europe).

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(*e*) Clearly the SP-2T results are much inferior to the UNORP results. This was to be expected as the SP-2T program does not use an 'orbit relaxation' solution, but is a pure 'translocation' program. This is highlighted by the relatively good results in the columns corresponding to weeks 1 and 3 (averaging around 2 m), and the poor results (averaging 7 m) in the 'troubled' week 2 (see §4.1).

(*f*) Further tests (not presented in this paper) show that, when the Doppler data are processed by using the 'orbit relaxation' technique, the use of the precise ephemeris (if made available) does *not* improve the results. On the contrary, it may even make them worse on account of the significant reduction in the amount of observational data (only two satellites).

5. Comparisons over the Nottingham triangle

In 1976 Doppler data were collected in the translocation mode by three JMR-1 receivers at three pillar stations near Nottingham. One of these stations belonged to the OSGB 70 scientific network and the other two were connected to it by observations of a primary geodetic standard. The triangle thus formed had sides of 48.5, 42.5 and 38.7 km respectively.

Table 3. Doppler – OSGB 70 chord distance differences (metres) for the Nottingham triangle

program	line 1	line 2	line 3	mean $\lvert\Delta\rvert$
UNORP	−0.57	+0.44	+0.09	0.37
SAGA	−0.58	+0.19	+0.93	0.56
SP-2T	−2.74	+1.68	+0.87	1.76

These data, which had already been processed by two commercial programs, JMR's SP-2T and DBA's SAGA, have also been processed by using Nottingham's UNORP program. Before carrying out any comparisons, the three terrestrial distances were corrected for a known systematic scale error of 3×10^{-6} in the OSGB 70 system (Ashkenazi, Crane, Williams & Dean 1979, this symposium). Considering the shortness of the lines involved, no corrections were applied to the Doppler derived coordinates.

The results of comparisons between the chord distances computed from the Doppler coordinates, on the one hand, and from the OSGB 70 coordinates (corrected for scale), on the other, are given in table 3. Clearly UNORP and SAGA are both good relative positioning programs, leading to mean values in distance difference of 37 and 56 cm respectively. The SP-2T program leads to a corresponding average of 1.76 m, which is all that can be expected from a pure translocation program, by using the broadcast ephemeris. The above results further confirm those found in §4.

6. Conclusions

(1) Nottingham's UNORP program seems capable of relative positioning accuracies averaging 30 cm over lines up to 300 km in length.

(2) For lines up to 600 km in length, accuracies appear to drop to an average of 60 cm. However, this may be a rather pessimistic assessment because of the weakness of the 'absolute standard' used in the comparison.

(3) It is essential to apply known transformation bias parameters to the raw broadcast ephemeris derived positions, especially over longer lines.

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(4) The availability of the precise ephemeris does *not* improve the relative positions derived from an orbit relaxation program.

(5) Considering the system errors in the broadcast ephemeris and its occasional vagaries (see §4.1), a relative positioning method based on an 'orbit relaxation' or a 'short arc' technique seems to be well suited, especially for shorter range work.

The work leading to this paper was completed in the Department of Civil Engineering of the University of Nottingham under the direction of Professor R. C. Coates.

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